Summary:

The alpha network is a network of parallel straight lines, able to cover the whole plane with a perfect central fivefold symmetry and possessing only four different lengths of line segments defined by the intersections of these straight lines except in the central zone called “black hole.”

This article consists in a presentation of the basis of a much more developed research on the Fibonacci sequence, plane tilings and fivefold symmetry available on my website. I also explain how, thanks to the discovery of a “mirror” architecture and its recurrence on an infinity of scales all along the infinite Fibonacci word, I finally determined with certainty the place of the origin (O.), starting point and center of symmetry of the Fibonacci sequence.

The alpha network possesses many geometrical particularities connected to the processes by which living organisms develop (mitosis), and probably with other phenomena. The network possesses four different states resulting from a symmetry breaking phenomenon briefly described in this article.
The α (alpha) network is a set of parallel straight lines having two different spacings of $\phi$ and $\phi^2$ proportions, propagating infinitely, and having accomplished five identical $36^\circ$ ($\pi/5$) rotations around an origin (O.) precisely determined, covering therefore the infinity of the plane in a fivefold symmetry, and possessing, on the whole plane (except for the central zone called “black hole”), only four segment lengths without intersections with the different lines of the network.
This network seems, at first sight, simple and trivial; however maintaining the coherence of such a network, and most of all a perfect fivefold symmetry on the whole plane, implies a flawless logic. Unlike periodic 2,3,4,6 fold networks that require even numbers for their constructions, fivefold symmetry necessarily refers to the golden ratio of its different segment lengths; $\phi$ (phi) is an irrational number that, consequently, possesses infinite decimals:

$$\phi = \frac{(1 + \sqrt{5})}{2} \approx 1.6180339887498948...$$

We therefore understand how delicate, or even impossible, it is to build any structure on long distances using such ratios.

The $\alpha$ (alpha) network evens itself out on the whole plane; it contains only intersections formed of only two straight lines, four different segments without intersections, and a perfect fivefold central symmetry, what gives peculiar characteristics to it. This small miracle is made possible by two things:

- The development of the “Fibonacci sequence” and of the “infinite Fibonacci word.”
- The precisely determined origin (O.), center of symmetry.

The infinite Fibonacci word, which I called ($F_{(0)}$) in my research, is constructed by taking two letters from any alphabet and by respecting substitution rules, such as:

Starting at [L] (first term), then [L,S] (second term), the following terms being obtained by replacing [L] of the previous term with [L,S], then [S] of the previous term with [L], what gives for the third term: [L, S, L], for the forth: [L,S,L,L,S]; for the fifth: [L,S,L,L,S,L,S,L,L]; etc…. endlessly.

In a more conventional way, we consider that the “Fibonacci words,” which constitute the “infinite Fibonacci word,” are to concatenation as Fibonacci numbers are to addition.

If we set the alphabet on L and S, the sequence ($S_n$) of the Fibonacci numbers is defined by $S_1 = S$ and $S_2 = L$ and for $n > 2$, by: $S_n = S_{n-1} \cdot S_{n-2}$ the product is the concatenation of the two previous terms, in other words, $S_n$ is obtained by juxtaposing $S_{n-1}$ and $S_{n-2}$.

The “infinite Fibonacci word,” written $S_{\infty}$, is the limit of this sequence, that is to say, the unique infinite word for which all words $S_n$ are prefixes. The Fibonacci words are so called by analogy with the Fibonacci numbers, since the construction process is analog, by replacing addition with concatenation.
The first Fibonacci words are:

\[ S_1 = S \]
\[ S_2 = L \]
\[ S_3 = LS \]
\[ S_4 = LSL \]
\[ S_5 = LSLLS \]
\[ S_6 = LSLLSLSL \]
\[ S_7 = LSLLSLSLLSLLS \]
\[ S_8 = LSLLSLSLLSLSLLSLSL \]

Whatever the method used, the infinite Fibonacci word begins as follows:

\[ \text{LSLLSLSLLSLSL} \ldots \infty \]

The “Fibonacci sequence” made up of the Fibonacci numbers begins as follows:

\[ F_0 = 0 ; F_1 = 1 ; F_2 = 1 ; F_3 = 2 ; F_4 = 3 ; F_5 = 5 ; F_6 = 8 ; F_7 = 13 ; F_8 = 21 ; F_9 = 34 ; \text{etc...} \]

and we can therefore verify the recurrence: \( F_n = F_{n-1} + F_{n-2} \)

The “infinite Fibonacci word” and the “Fibonacci sequence” possess many physical and mathematical properties that have been described at length in books on the subject; but there are some properties I have never heard of before and that makes me think that they might not have been noticed yet.

We observe, for example, that the first word \( (S_1) \) is not part the infinite Fibonacci word anymore, as if it had been naturally excluded from it.

The Fibonacci sequence also displays a kind of “vagueness” in the introduction as soon as one wants to adapt it to the description of a real physical construction. For example, what are the first terms to describe the proliferation of a rabbit population? A rabbit cannot be born from a zero, neither from a one by the way; there need to be a first rabbit, and this rabbit has to meet a second rabbit for the species to start proliferating. In the same way, when one wants to form the infinite Fibonacci word or make a sequence of spaced parallel straight lines, large for \[ L \] and narrow for \[ S \], which one should be placed first, and where should the origin be? Is there a categorical answer to these questions?

It amounts to confronting oneself to the mystery of creation…
When one observes the infinite Fibonacci word, that I called \( (F_{(0)}) \) in my research, one can see that it is composed of three different modules whose succession in the whole word forms a superior sequence which I called \( (F_{(1)}) \). These three different subsequences are:

- \( b_1 = \text{LSLLSLSL} \) of \( F_{(0)} \)
- \( d_1 = \text{LSLSLLSL} \) of \( F_{(0)} \)
- \( c_1 = \text{SL} \) of \( F_{(0)} \)

They are going to succeed one another in the whole infinite Fibonacci word, revealing a very singular structure:
\(F_{(1)}\) will therefore be a new infinite word formed of the three letters: b, d, c, corresponding to these three different modules and covering \(F_{(0)}\) according to this structural logic:

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bbddcc blddc bdddc bdddc bdddc
bbddcc bdddc bdddc bdddc bdddc
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etc...

We observe:

- that modules b and d are mirrors as far as the spacings of \(F_{(0)}\) they contain are concerned, and that they only differ by their middle spacings, [LS] or [SL], the other spacings
of these modules are distributed symmetrically on each side of the middle spaces. This is important since we are looking for symmetry.

- that the structural logic of the word $F_{(1)}$ is repeated at a greater scale since the three different arrows in the margins of the development succeed one another and cover individually the same sequence of [L] and [S] spacings of $F_{(0)}$, and the same sequence of $b_1$, $d_1$, and $c_1$ modules of $F_{(0)}$.

However, we can check that, to find the same structural logic of repartition of the spacings of $F_{(0)}$ in $b$, $d$, $c$ modules of a greater scale (that is to say with two spacings [LS] or [SL] centered in the module, then a symmetrical distribution of the other spacings, on each side of the central spacings), we will have to give up the first three letters of the word $F_{(0)}$: [LSL].

These modules that we will call $b_2$, $d_2$, and $c_2$ will compose the word $F_{(2)}$, identical to $F_{(1)}$; $b_2$ and $d_2$ will be mirrors and will only differ by their two different central spacings, [SL] or [LS], they will contain 34 spacings of $F_{(0)}$; $c_2$ will contain 8 spacings of $F_{(0)}$ and will identical to $b_1$.

\[
b_2 = \text{LSLSLLSLLSLLSLSLSLLSLLSLLSLLSLSLSLLSLLSL}
\]

\[
d_2 = \text{LSLSLLSLLSLLSLSLSLSLLSLLSLSLSLSLSLSLSLSL}
\]

\[
c_2 = \text{LSLLSLSL}
\]

therefore: $b_2 = d_1d_1c_1b_1b_1$ ; $d_2 = d_1d_1c_1b_1b_1$ ; $c_2 = b_1$

We observe:

- that, in order to describe the contents of a $d_2$ module correctly, we have to consider a new module called $\omega_1$ (mirror $c_1$), containing [LS] of $F_{(0)}$. 
that after giving up [LSL] at the start, we will be able to build $F_{(0)}$ again by making $b_2$, $d_2$ and $c_2$ succeed one another, according to the $F_{(1)}$ and $F_{(2)}$ patterns.

- that there is also a mirror effect between $F_{(1)}$ and $F_{(2)}$ since the two middle spacings [LS] in $b_1$ become [SL] in $b_2$; the middle spacings [SL] in $d_1$ become [LS] in $d_2$; and the middle spacings [SL] in $c_1$ become [LS] in $c_2$.

We can also verify that, to find the same structural logic in the repartition of the spacings of $F_{(0)}$ in the $b$, $d$, $c$ modules at an even greater scale, we will at first have to give 16 spacings of $F_{(0)}$ up, that we will then have a word $F_{(1)}$ identical to $F_{(1)}$ and $F_{(2)}$, formed of three modules: $b_3$, $d_3$, and $c_3$; and that we will therefore be able to rebuild $F_{(0)}$ after having given [LSLLSLLSLLSLLSL] up at the beginning of the word with these three new modules, and, by making them succeed one another according to the patterns of $F_{(1)}$; $F_{(2)}$ and $F_{(3)}$. $b_3$ and $d_3$ will contain 144 spacings of $F_{(0)}$; $c_3$ will contain 34 spacings of $F_{(0)}$.

Finally we notice that the structural logic described previously, governed by the Fibonacci sequence, will allow us to consider all the words $F_{(n)}$ at greater scales. Actually, the same word repeats itself over and over at all scales. If we give to $[S]$ and $[L]$ the respective values: $[S] = \phi = (1 + \sqrt{5})/2$; and $[L] = \phi^2 = ((1 + \sqrt{5})/2)^2$, the word $F_{(n)}$ with $(n)>0$, will repeats itself at all $\phi^3$ scales with a mirror effect at every of these scales.

But the most interesting thing is that the words $F_{(n)}$ with $(n)>1$ have to be shifted from the origin (first spacing of $F_{(0)}$) to find the same structural logic. This phenomenon is undoubtedly described by the Fibonacci numbers.
Let's see what the following table can tell us:

Descriptive table of the spacings of $F_{(0)}$ contained in the different $b_n$, $d_n$, $c_n$ modules of $F_{(n)}$ scales, and relation with the Fibonacci numbers.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$F_n$</th>
<th>$c_n$ = 2 esp.</th>
<th>$b_n$; $d_n$ = esp.</th>
<th>$n$ esp. and</th>
<th>$n'$ esp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>196418</td>
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</tbody>
</table>

etc...
We notice before starting that:

- \( F_2 \) leaves 3 spacings of \( F_0 \); i.e. [LSL].
- \( F_3 \) leaves 3 + 13 spacings of \( F_0 \); i.e. [LSLLSLLSLLSLSL]
- \( F_4 \) leaves 3 + 13 + 55 = 71 spacings
- \( F_5 \) leaves 3 + 13 + 55 + 233 = 304 spacings
- \( F_6 \) leaves 3 + 13 + 55 + 233 + 987 = 1291 spacings
- \( F_7 \) leaves 3 + 13 + 55 + 233 + 987 + 4181 = 5472 spacings
- \( F_8 \) leaves 3 + 13 + 55 + 233 + 987 + 4181 + 17711 = 23183 spacings
- etc …

We can therefore wonder:

Why is \( F_1 \), not shifted from \( F_0 \)?...

And we observe:

- For \( F_2 \): 1 + 3 = 4 = 8/2 = \( c_2 /2 \)
- For \( F_3 \): 1 + 3 + 13 = 17 = 34 / 2 = \( c_3 /2 \)
- For \( F_4 \): 1 + 3 + 13 + 55 = 72 = 144 / 2 = \( c_4 /2 \)
- For \( F_5 \): 1 + 3 + 13 + 55 + 233 = 305 = 610 / 2 = \( c_5 /2 \)
- For \( F_6 \): 1 + 3 + 13 + 55 + 233 + 987 = 1292 = 2584 / 2 = \( c_6 /2 \)
- etc...

We notice that, by adding 1 to the number of spacings of \( F_0 \) that every \( F(n), (n>0) \) has to give up in order to start their development, we find the number of spacings of \( F_0 \) contained in every of the \( c_{(n)} \) modules divided by two.
Thus, one spacing is missing at the introduction of $F_{(0)}$!

What metrical value does this (1) have in $F_{(0)}$?

Given that we have to give up a spacing value from the origin, equal to $c_{(n)}/2$ in order to obtain the “mirror effect” between $b_{(n)}$ and $d_{(n)}$ of $F_{(n)}$, we will have to calculate the metric value of the spacing that introduces $F_{(0)}$:

$$C_{(1)}/2 = SL/2 \text{ given that: } S = \phi \text{ and } L = \phi^2$$

Gives $$(\phi + \phi^2)/2 = \phi^3/2 \sim 2.11803398875…$$

And: $$\phi^3/2 = \phi + 0.5 = \phi^2 - 0.5$$

We have, thanks to this rather simple, located the exact place of “the origin (O.)” introducing $F_{(0)}$; $F(n)$, $(n>0)$; the Fibonacci sequence and the “infinite Fibonacci word.”

Given that we have been considering from the introduction of this research that the origin (O.) was probably the center of symmetry of a hypothetical “supersymmetry,” everything that develops in a direction from that origin, will also have to develop in the opposite direction. This means that $F_{(0)}$ should start this way:

\[
\begin{array}{cccccccccccccccc}
\text{...} & 1 & 0 & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & 144 & 233 & 377 & 609 & 986 & 1595 & \text{...}
\end{array}
\]

The origin (O.) is therefore perfectly centered in an original space corresponding to: $c_1$; or $\varphi_1$; or [SL]; or [LS]; and with a $\phi^3$ value when $S = \phi$ and $L = \phi^2$. 
This configuration explains the “imprecision” at the introduction of the Fibonacci sequence, because the very first spacing separating the origin (O.) from the first straight line does not correspond either to [L] or to [S], but rather to [LS] / 2 and thus no straight line passes through the origin (O.).

When $S = \phi$ and $L = \phi^2$; if we introduce the sequence with a spacing $S$ instead of $\phi^3 / 2$, a straight line would be found on the right of the origin (O.) at a (+ 0.5) distance; and if we introduce it with a spacing $L$ instead of $\phi^3 / 2$, a straight line would be found on the left of the origin (O.) at a (-0.5) distance.

This straight line, which can be found at the very best at (+ 0.5) or (- 0.5) from the origin (O.) with the same probability, generates a symmetry breaking phenomenon. We will call this line: “hesitant straight line” (“Droite hésitante (Dh)”).

Taken from Frédéric Mansuy’s research on fivefold symmetry.

If you do not want this journey to end, you can visit my website: http://supersymetrie.fr
Fibonacci Word and Symmetry Breaking

(Taken from my research: Supersymétrie d’ordre cinq périodique et effet miroir dans la suite de Fibonacci (Periodic Fivefold Supersymmetry and Mirror Effect in the Fibonacci Sequence) See: http://supersymetrie.fr)

When [S] and [L], which compose the infinite Fibonacci word, represent two different spacings in a network of parallel straight lines, if [S] = ϕ = (1 + √5)/2 and [L] = ϕ² = ((1 + √5)/2)², the origin (O.) is to be found at [LS]/2, that is to say ϕ³/2 = 2.1180339887..., a straight line (which I named “droite hésitante” (Dh), (hesitant straight line)) can therefore with the same probability pass at a (+ 0.5) or (-0.5) distance from the origin (O.), as the pendulum of a clock would. This uncertainty of position creates a potential movement of amplitude (1), and therefore an energy. The central zone made up of two spaces [S] + [L] of a ϕ³ value becomes inconsistent since it could either be [SL] or [LS], hence the symmetry breaking phenomenon; it thus represents a singularity which concentrates the energy of the (1) movement; one could think about a black hole.

We observe that the Fibonacci word, which I called F_{(0)} in my research, is perfectly covered by another infinite word at every F_{(n)} (n>0) scale; that this superior word undergoes a left-right alternation at every of these scales with the origin (O.) as center of symmetry. This means that for all the even values of (n), the word will be identical, and that for all the uneven values (n), the word will be “mirror” identical.
When \([S] = \phi = (1 + \sqrt{5})/2\) and \([L] = \phi^2 = ((1 + \sqrt{5})/2)^2\), the word \(F_{(n)}\) \((n > 0)\) repeats itself with an inversion (mirror effect) at all \(\phi^3\) scales, this means that there is a recurrence of the word at every \(\phi^6\) scale.

Infinite word \(F_{(n)}\) with \((n > 0)\) covering the Fibonacci word:

The right part of the word becomes left and inversely at every \(\phi^3\) scale. This word which superposes itself to the infinite Fibonacci word is therefore recurrent at every \(\phi^6\) scale when \([S] = \phi = (1 + \sqrt{5})/2\) and \([L] = \phi^2 = ((1 + \sqrt{5})/2)^2\).

For further information, see my website: http://supersymetrie.fr
Conclusion:

The alpha network and the symmetry breaking phenomenon described in this article are the key that unlocks the door to a geometrical, symmetrical, still unexploited and particularly harmonious world. The omnipresence and the predominance of symmetries in all science fields are now established facts, and plane tilings that superimpose themselves perfectly on the alpha network could solve many enigmas that still puzzle scientists, in domains as diverse as mathematics, quantum physics, chemistry, biology, astrophysics, crystallography (quasi crystals)"

Unlike Penrose tilings that possess no extended underlying structural logic, the alpha network offers a solid, coherent architecture to fivefold symmetry and a development implacably organized by the infinite Fibonacci word and the golden ratio on the whole plane.

For further information, see my website: http://supersymetrie.fr